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# RENEGING IN QUEUES WITHOUT WAITING SPACE

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## ABSTRACT

Reneging involves the phenomenon of customers joining a queuing system but departing without completely receiving service. There are situations where customers begin receiving service but disengage before service completion. Emergency patients attended by critical care professionals are of such type. This paper deals with the analysis of Markovian finite buffer queuing system without any waiting space under the assumption that customers may disengage i.e. renege before the completion of service. Explicit results for this model are presented. A few fresh performance measures have been suggested. A numerical example with design implications rounds up the paper.

**KEYWORDS:** Queuing, Reneging, Finite Buffer, Customer Impatience, Patience Time

## INTRODUCTION

This paper deals with the analysis of a multiserver Markovian queuing system with finite buffer and no waiting space. Arriving customers either go straight into service or are refused entry into the queuing system. We analyze it under the assumption that customers who arrive into the queuing system cannot stay on infinitely till completion of their service and may renege in between. This model has been dealt with in literature but without the assumption of impatient customers. The importance of the assumption spells from the fact that in case of such customers, closed form useable results for this model are still not available.

The particular interest in this paper is reneging behavior of customers. In Queuing literature, a customer is said to have reneged if it leaves the system without receiving its service entirely. In our day-to-day life, reneging is a commonly observed phenomenon. People join a queue and in the process of waiting – either in the queue or while receiving service – get impatient and leave the system. To any system manager, the implications are unfavorable as reneging implies loss in business in addition to an additional dissatisfied customer. In our competitive world, a business loss implies revenue loss. In addition, a dissatisfied customer is often likely to spread the word which has ripple effect. While a queuing system without reneging is a wishful thought, nevertheless it is an inevitable part of any queuing system.

Broadly speaking, reneging is of two types- viz. reneging till beginning of service (henceforth referred to as R_BOS) and reneging till end of service (henceforth referred to as R_EOS). In R_BOS, a customer can renege only as long as it is in the queue. It cannot renege once it begins receiving service. A common example is the barbershop. A customer can renege while he is waiting in queue. However once service get start i.e. hair cut begins, the customer cannot leave till hair cutting is over. On the other hand, in R_EOS, a customer can renege not only while waiting in queue but also while receiving service. Such a situation may occur in the processing or merchandising of perishable goods, hospital emergency room/O.T. handling critical patients etc.

Even though queuing models with varied assumptions have been analyzed by queuing theorists, reneging has not found a mention in most of such work. Given its importance in the sense that it is a commonly observed phenomenon, this is somewhat ironical. In this paper, we attempt to model this phenomenon in the Markovian finite buffer queuing system without waiting space, symbolically denoted by M/M/c/c. In this model, an arriving customer leaves the system without waiting for service if all the c channels are busy on its arrival. Otherwise it goes straight into service. It is also called a c-channel loss system (Medhi, 2003). To the best of our knowledge, an explicit analysis of this model with reneging customers has not been carried out. This forms the motivation of our work.

Customers who are allowed to enter the system begin receiving service immediately. It is therefore obvious that the reneging rule in the queuing model can only be of R_EOS type. For the purpose of our analysis, we shall assume that $\lambda$ and $\mu$ are the arrival and service rates. As regards reneging, each customer will be assumed to have a reneging distribution following exp ($\nu$). Considering the system as a whole, reneging from the system, it will be a function of the system state. If the system in the state 'n' ($n \leq c$), the reneging rate from the system will be $n\nu$.

The rest of the paper is structured as follows. In section 2, we provide a review of literature. In section 3, steady state probabilities are presented. In section 4 some non-traditional performance measures are derived. Sensitivity analysis is carried out in section 5. A numerical example is discussed in section 6. Section 7 concludes the paper. Some derivations are given in appendix.

## LITERATURE SURVEY

One of the earliest works on reneging was by Barrer (1957a) where he considered deterministic reneging with single server Markovian arrival and service rates. Customers were selected randomly for service. In his subsequent work, Barrer (1957b) also considered deterministic reneging (of both R_BOS and R_EOS type) in a multiserver scenario with FCFS discipline. Another early work was by Haight (1959) where he considered a queue in which a person having joined may decide to leave and give up service if it appears that the time consumed will exceed some maximum which he has available. Ancker and Gafarian (1963a) carried out an early work on Markovian reneging with Markovian arrival and service pattern. They assumed that customer arrived from a single infinite source in a Poisson stream, arriving customer may balk with probability n/N where 'n' is the total number of customer in the system and 'N' is the maximum number of customer allowed in the system. For steady state, they obtained the steady state probabilities, mean number in queue and system, probability of balking, waiting, reneging, acquiring service and customer loss rate. All of these results are also obtained for a pure balking system (no reneging) by setting reneging parameter equal to 0. Ancker and Gafarian (1963b) in their next paper considered balking and reneging together but here balking rate is drastically altered. They assumed that arriving customers join the system if it is empty or balk with probability $1-\beta/n$; n=1, 2, …, n where 'n' is the number of customers in the system and $\beta$ is the measure of willingness to join the queue. The other assumption and derivations are similar to the previous paper. Ghosal (1963) also considered a single server queuing system in which a customer does not wait more than a fixed time k, so that if he does not get his service within this time, he departs. Some results are obtained by applying the theory of storage. Gavish and Schweitzer (1977) also considered a deterministic reneging model with the additional assumption that arrivals can be labeled by their service requirement before joining the queue and arriving customers are admitted only if their waiting plus service time do not exceed some fixed amount. This assumption is met in communication systems. Baccelli et al. (1984) considered GI/GI/1 queue with general reneging where they established the extension of classical GI/GI/1 formulae concerning the stability condition and the relation between actual and virtual waiting time distribution function. Kok and Tijms (1985) considered a single server queuing system where a customer becomes a lost customer when its service has not begun within a fixed time.

Haghighi et al. (1986) considered a Markovian multi-server queuing model with balking as well as reneging. Each customer had a balking probability which was independent of the state of the system. Reneging discipline considered by them was R_BOS. Liu et al (1987) considered an infinite server Markovian queuing system with reneging of type R_BOS. Customers had a choice of individual service or batch service, batch service being preferred by the customer. Martin and Artalejo (1995) considered an M/G/1 model with two types of impatient units. The arrival rate of type I customers followed Poisson law with parameter $\lambda_1$ and service pattern follows general distribution with probability generating function $B_1(x)$. If the server is busy on arrival of a customer, he immediately left the system i.e. of balking type. For the second type of customers it is assumed that arrival rates follow Poisson law with parameter $\lambda_2$ and service pattern follows general distribution with probability generating function $B_2(x)$. On arrival of a customers, if the server is busy the second type of customers enter into the system and wait to be proceed later on i.e. of reneging type. They developed an exhaustive analysis of the system including embedded Markov chain, fundamental period and various classical stationary probability distributions. More specifically, performance measures such as the number of lost customers and other quantities were also considered. The mathematical analysis of the model was based on the theory of Markov renewal processes. Shawky (1997) considered a single server machine interference model with balking, reneging and an additional server for longer queues. Service discipline was considered as FIFO. The steady state probabilities and some measures of effectiveness were derived in an explicit form. Finally, some particular cases for the multi-server models were deduced. Brandt et al. (1999) considered a S-server system with two FCFS queues, where the arrival rates at the queues and the service may depend on number of customers 'n' being in service or in the first queue, but the service rate was assumed to be constant for n>s. The customers in the first queue were assumed impatient customers with deterministic reneging. Boots and Tijms (1999) considered an M/M/C queue in which a customer leaves the system when its service has not begun within a fixed interval after its arrival. In this paper, they have given the probabilistic proof of 'loss probability', which was expressed in a simple formula involving the waiting time probabilities in the standard M/M/C queue.

Wang et al (1999) considered the machine repair problem in which failed machines balk with probability (1-b) and renege according to a negative exponential distribution. Another work using the concepts of balking and reneging in machine interference queue has been carried out by Al-Seedy and Al-Ibraheem (2001). Bae et al. (2001) considered an M/G/1 queue with deterministic reneging. They derived the complete formula of the limiting distribution of the virtual waiting time explicitly. Choi et al. (2001a) introduced a simple approach for the analysis of the M/M/C queue with a single class of customers and constant patience time by finding simple Markov process. Applying this approach, they analyzed the M/M/1 queue with two classes of customer in which class 1 customer have impatience of constant duration and class 2 customers have no impatience and lower priority than class 1 customers. Performance measures of both M/M/C and M/M/1 queues were discussed. Choi et al. (2004b) considered a multi server Markovian queue with deterministic reneging. They obtained a simple Markov process by using a concept of virtual waiting time and then obtained the stationary distribution of the Markov process. Different performance measures such as loss probability, waiting time distribution, mean waiting time and mean queue size have been calculated by using the results of the stationary distribution of Markov process. Zhang et al. (2005) considered an M/M/1/N framework with Markovian reneging where they derived the steady state probabilities and formulated a cost model. Some performance measures were also discussed. A numerical example was discussed to demonstrate how the various parameters of the cost model influence the optimal service rates of the system. Singh et al. (2007) dealt with a single server Markovian queuing model with controllable arrival rates with discouragement factor reneging in which it is assumed that arrival and service processes are interdependent. The stationary state solutions of the model are analyzed here. The expression for system characteristics as average number of customers in the system and average waiting times are determined. The numerical illustrations are also considered to validate the analytical results and to illustrate the effect of the parameters on several performance characteristics.

Choudhury (2008) analyzed a single server Markovian queuing system with the added complexity of customers who are prone to giving up whenever its waiting time is larger than a random threshold-his patience time. He assumed that these individual patience times were independent and identically distributed exponential random variables. A detailed and lucid derivation of the distribution of virtual waiting time in the system was presented. Some performance measures were also presented. El- Paoumy (2008) also derived the analytical solution of Mx/M/2/N queue for batch arrival system with Markovian reneging. In this paper, the steady state probabilities, some performance measures of effectiveness were derived in explicit forms. Another paper on Markovian reneging was by Yechiali and Altman (2008). They derived the probability generating function of number of customers present in the system and some performance measures were also discussed. Xiong et al. (2008) considered a single server queue with a deterministic reneging time motivated by the timeout mechanism used in application servers in distributed computing environments. They had employed a Volterra integral equation to study the M/G/1 queue with reneging using level crossing analysis. They derived the probability generating function of number of customers present in the system and some performance measures were discussed. El-Sherbiny (2008) also considered a non-truncated MX /M/1 queue with reneging, balking, state-dependent and an additional server for longer queues to derive the solution of the queue. Here he assumed that the units arrived in batches of size X which was a random variable and queue discipline considered was the usual one ‘FIFO’. In this paper, the researcher investigated the probability generating function of the number of units in the system and some special cases were also deduced. Jouini et al. (2009) considered two multi-class call center models with and without reneging. They assumed that customers had different priorities and the content of different types of calls was assumed as similar allowing their service times to be identical. Choudhury (2009) considered a single server finite buffer queuing system (M/M/1/K) assuming reneging customers. Both rules of reneging were considered and various performance measures presented under both rules of reneging. Xiong and Altiok (2009) studied a multi-server queue with Poisson arrivals general service time distribution and deterministic reneging times. Via approximations, they provided the expression for mean waiting time. This work was motivated by the time-out mechanism used in managing application servers in transaction processing environments. El-Paoumy and Ismail (2009) considered an Mx/Ek/1/N model with balking and reneging in which they assumed that units arrive in batches of random size ‘X’ with the inter-arrival times of batches following negative exponential distribution, the queue discipline is FCFS, it also assumed that batches are pre-ordered for service purpose, the service time distribution is Erlangian with k-stages, there is only one server and system capacity is restricted to ‘N’. Recurrence relations connecting the various probabilities introduced were calculated. Some measures of effectiveness were deducted and some special cases were also obtained.

## THE STEADY STATE PROBABILITIES

In this section, the steady-state probabilities are derived by the Markov process method. Henceforth, the reneging rule considered would be R_EOS. Let pn denotes the probability that there are ‘n’ customers in the system in steady-state. Applying the Markov process theory, we obtain the following set of steady- state equations.

$$\lambda\ p_0 = (\mu + \nu)\ p_1 \tag{1}$$

$$\lambda p_{n-1} + (n+1)(\mu+\nu) p_{n+1} = \lambda p_n + n(\mu+\nu) p_n \quad ; 1 \le n \le c-1 \tag{2}$$

$$\lambda\ p_{c-1} = c(\mu + \nu) p_c \tag{3}$$

Solving recursively, we get

$$p_n = \frac{\lambda^n}{n!(\mu+\nu)^n} p_0 \quad ; \text{n=1, 2… c}$$

where $p_0$ is obtained from the normalizing condition $\sum_{n=0}^{c} p_n = 1$ and is given as

$$p_0 = \left[1 + \sum_{n=1}^{c} \frac{\lambda^n}{n!(\mu+\nu)^n}\right]^{-1}$$

$$= \left[\sum_{n=0}^{c} \frac{\lambda^n}{n!(\mu+\nu)^n}\right]^{-1} \tag{4}$$

It represents the probability that all the servers are idle. Then

$$p_n = \frac{\left(\frac{\lambda}{(\mu+\nu)}\right)^n / n!}{\sum_{n=0}^{c} \left(\frac{\lambda}{(\mu+\nu)}\right)^n / n!}$$

which is the analogue of Erlang's first formula under reneging since an arriving unit who finds all channels busy leaves the system. The probability of this event is

$$p_c = \frac{\lambda^c}{(\mu+\nu)^c c!} p_0$$

which is modified Erlang's loss formula or blocking formula for reneging. We shall denote it by $B_R(c, \lambda/\mu+\nu)$

## PERFORMANCE MEASURES

Performance measures are generally the specific representation of a capacity, process or outcome deemed relevant to the assessment of performance, which are quantifiable and can be documented. The main objective of any queuing study is to assess some well-defined parameters, which are designed at striking a good balance between customer satisfaction and economic considerations. In queuing theory, measures through which the nature of the quality of service can be studied are known as performance measures. Performance measures are important as issues or problems caused by queuing situations are often related to customer's dissatisfaction with service or may be the root cause of economic losses in a business. Analysis of the relevant performance measures of queuing models allows the cause of queuing issues to be identified and the impact of proposed changes to be assessed. Some of the performance measures of any queuing system that are of general interest for the evaluation of the performance of an existing queuing system and to design a new system in terms of the level of service a customer receives as well as the proper utilization of the service facilities include mean size, server utilization, customer loss and the like.

An important performance measure is 'L' which denotes the mean number of customers in the system. To obtain the expression for the same, we note that,

$$L = P'(1) = \frac{d}{ds} P(s)\,|_{s=1}$$

where P(s) is the p.g.f. of the steady state probabilities. The derivation of P′(1) is given in the appendix. Then from (A.1)

$$L = \lambda(1 - p_c)/(\mu + \nu)$$

Variance of the number of customers in the system has also been derived. Then from (A.2) we have,

$$V(N) = \frac{\lambda}{(\mu+\nu)}\left[\frac{\lambda(1-p_c)}{(\mu+\nu)} - cp_c\right]$$

Mean system size for the particular case with no reneging can be similarly derived by reconstructing the steady state equation. In that case we have,

$$L_{(\nu=0)} = \lambda(1 - p_c)/\mu$$

Customers arrive into the system at the rate of λ. However all the customers who arrive do not join the system because of finite buffer restriction. The effective arrival rate into the system is thus different from the overall arrival rate and is given by

$$\begin{aligned}\lambda^e &= \lambda p_0 + \lambda p_1 + \ldots + \lambda p_{c-1} + 0.p_c \\ &= \lambda \sum_{n=0}^{c-1} p_n \\ &= \lambda(1 - \mathrm{p_c})\end{aligned} \qquad (5)$$

Where

$$p_c = \frac{\lambda^c}{c!(\mu+\nu)^c}\, p_0$$ '$p_0$' is given in (4)

We have assumed that each customer has a random patience time following exp (ν). Clearly then, the reneging rate of the system would depend on the state of the system. The average reneging rate (avg rr) is given by

$$\begin{aligned}Avg\ rr &= \sum_{n=1}^{k} n\nu p_n \\ &= \nu p'(1) \\ &= \lambda\nu(1 - p_c)/(\mu+\nu)\end{aligned} \qquad (6)$$

In system management, customers who renege represent business lost. It is therefore of interest to determine the proportion of customers lost, both out of these joining the system as well as out of those arriving into the system. These are given below

Proportion of customer lost due to reneging out of those arriving and joining the system is

$$= Avgrr/\lambda^e$$

$$= \lambda\nu/(\mu+\nu)$$

using (5) and (6)

Proportion of customer lost due to reneging out of total customers arriving in the system is using (6)

$$= Avgrr/\lambda$$
$$= \nu(1-p_c)/(\mu+\nu)$$

In totality, customers are lost to the system in two ways, due to finite buffer and due to reneging. The management would like to know the proportion of total customers lost in order to have an idea of total business lost. Hence the mean rate at which customers are lost is

Rate of loss due to finite buffer+ Avgrr

$$= \lambda - \lambda^e + Avgrr$$

$$= \{\lambda\nu(1-p_c)/(\mu+\nu)\} + \lambda p_c \quad \text{using (5) and (6)}$$

$$= \{\lambda\text{-}\lambda^{\text{e}} + avgrr\}/\lambda$$
$$= \nu + \mu p_c/(\mu+\nu)$$

This rate helps in the determination of proportion of customers lost which is

$$\begin{aligned} &= \{\lambda\text{-}\lambda^{\text{e}} + avgrr\}/\lambda \\ &= \nu + \mu p_c/(\mu+\nu) \end{aligned} \tag{7}$$

The proportion of customers completing service is its complement.

## SENSITIVITY ANALYSIS

We have assumed that there are essentially four parameters viz: $\lambda, \mu, \nu$ and $c$ relating to the stochastic nature of arrival, service reneging patterns and system capacity. Various reasons may influence these parameters so that on different occasions these may undergo change. From managerial point of view, an idle server is a waste. Similarly low server utilization is also a waste. It is therefore interesting to examine and understand how server utilization varies in response to change in system parameters. We place below the effect of change in these system parameters on server utilization. For this purpose, we shall follow the following notational convention in the rest of this section.

Let $p_n$ ($\lambda, \mu, \nu$ ,c) denote the probability that there are ‘n’ customers in a system with parameters $\lambda, \mu, \nu, c$ in steady state under R_EOS.

It can be shown that

- Let $\lambda_1 > \lambda_0$ then

$$\frac{p_0(\lambda_1,\mu,\nu,c)}{p_0(\lambda_0,\mu,\nu,c)} < 1$$

$$\Rightarrow \frac{(\lambda_0-\lambda_1)}{(\mu+\nu)} + \frac{(\lambda_0^2-\lambda_1^2)}{2!(\mu+\nu)^2} + \ldots$$
$$+ \frac{(\lambda_0^c-\lambda_1^c)}{c!(\mu+\nu)^c} < 0$$

Which is true and hence $p_0 \downarrow as\ \lambda \uparrow$.

- Let $\mu_1 > \mu_0$ then

$$\frac{p_0(\lambda,\mu_1,\nu,c)}{p_0(\lambda,\mu_0,\nu,c)} > 1$$

$$\Rightarrow \lambda\left(\frac{1}{(\mu_0+\nu)} - \frac{1}{(\mu_1+\nu)}\right) + \frac{\lambda^2}{2!}\left\{\frac{1}{(\mu_0+\nu)^2} - \frac{1}{(\mu_1+\nu)^2}\right\} +$$

$$\ldots + \frac{\lambda^c}{c!}.\left\{\frac{1}{(\mu_0+\nu)^c} - \frac{1}{(\mu_1+\nu)^c}\right\} > 0$$

Which is true and hence $p_0 \uparrow \quad as\ \mu \uparrow$

- Let $\nu_1 > \nu_0$ then

$$\frac{p_0(\lambda,\mu,\nu_1,c)}{p_0(\lambda,\mu,\nu_0,c)} > 1$$

$$= \lambda\left(\frac{1}{(\mu+\nu_0)} - \frac{1}{(\mu+\nu_1)}\right) + \frac{\lambda^2}{2!}\left\{\frac{1}{(\mu+\nu_0)^2} - \frac{1}{(\mu+\nu_1)^2}\right\} +$$

$$\ldots + \frac{\lambda^c}{c!}\left\{\frac{1}{(\mu+\nu_0)^c} - \frac{1}{(\mu+\nu_1)^c}\right\} > 0$$

which is true and hence $p_0 \uparrow as\ \nu \uparrow$.

Let $c_1 > c_0$ then

$$\frac{p_0(\lambda,\mu,\nu,c_1)}{p_0(\lambda,\mu,\nu,c_0)} < 1$$

$$\Rightarrow \sum_{n=1}^{c_0} \frac{\lambda^n}{n!(\mu+\nu)^n} - \sum_{n=1}^{c_1} \frac{\lambda^n}{n!(\mu+\nu)^n} < 0$$

which is true and hence $p_0 \downarrow as\ c \uparrow$

These results state that an increase in arrival rate would result in lowering of the fraction of time the server is idle. An increase in service rate would mean the server is able to work efficiently so that it can process same amount of work quickly. This translates to higher server idle time. An increase in reneging rate would mean the server has fewer work to do and hence higher fraction of idle time. If the number of server increased which may increase the arrival rate of customer would result in lowering of the fraction of time the server is idle.

## NUMERICAL EXAMPLE

To illustrate the use of our results, we apply them to a queuing problem. The problem we describe below has been suitably adopted from page 338 of Ravindran, Phillips and Solberg (1987). While we have not modified the system parameters, the set up has been changed to make it more relevant to the model in hand.

Consider the operation theatre (O. T.) of the emergency unit of a small hospital. The O.T. of this unit contains two beds, which are manned by doctors and paramedical staff round the clock. Since this is an emergency unit, there is no space for patients to wait. The state of the system is the number of patients in the O.T. 0, 1 and 2. If there is an empty bed when a customer arrives, he enters the O. T. and his treatment begins. If both the beds are occupied when he arrives, he does not enter the O. T. and leaves for another hospital. As soon as a patient's emergency treatment is over, he is shifted to the ward instantaneously. On the average, a patient arrives every 10 minutes and each patient takes an average of 15 minutes of time on the O. T. bed.

Notice that in the above problem, some potential patients are turned away. If the O. T. had another bed, it might be able to profit from additional paying patients. On the other hand, the additional facility (bed) would have to be paid. The hospital management would be interested to know if this additional investment would be worthwhile.

We assume Markovian arrival and service distribution. Further, since arriving patients who do not find the bed instantaneously leave the hospital, this is a finite buffer queuing system with no waiting space. Since the emergency patients arrive, it is possible that some patients would expire while being treated. This phenomenon would be a case fit for analysis using the concept of reneging till end of service. We assume two alternative scenarios. In the $1^{st}$ scenario mean reneging rate is assumed to be 1hour ($\nu$=1/hr) and in the $2^{nd}$ scenario the assumption is 30 minutes ($\nu$=2/hr).

Various performance measures of interest computed under the two reneging scenarios are given in Table 1 and 2. These measures were arrived at using a FORTRAN 77 program coded by the authors. The whole objective is to examine how the performance measures vary with the increase in bed capacity.

**Table 1: Performance Measures Assuming $\lambda$ =6/Hr, M=4/Hr, N=1/Hr**

| **Performance Measure** | **Number of Beds in O.T.** | |
|---|---|---|
| | **C=2** | **C=3** |
| Proportion of customers completing service. | 0.60274 | 0.72817 |
| Fraction of time that all server is idle ($p_0$) | 0.34246 | 0.31172 |
| Average length of system | 0.9041 | 1.09227 |
| Mean reneging rate | 0.9041 | 1.09227 |
| Rate of loss due to finite buffer. | 1.47945 | 0.53865 |
| Effective arrival rate | 4.52055 | 5.46135 |
| Proportion of customers lost (due to reneging and finite buffer) | 0.39726 | 0.27182 |

**Table 2: Performance Measures Assuming $\lambda$ =6/Hr, M=4/Hr, N=2/Hr**

| **Performance Measure** | **Number of Beds in O.T.** | |
|---|---|---|
| | **C=2** | **C=3** |
| Proportion of customers completing service. | 0.53333 | 0.625 |
| Fraction of time that all server is idle ($p_0$) | 0.4 | 0.375 |
| Average length of system | 0.8 | 0.9375 |
| Mean reneging rate | 1.6 | 1.875 |
| Rate of loss due to finite buffer. | 1.2 | 0.375 |
| Effective arrival rate | 4.8 | 5.625 |
| Proportion of customers lost (due to reneging and finite buffer) | 0.46667 | 0.375 |

In both the scenarios, we observe that with increase in number of beds, the proportion of customers who complete receiving services go up. It goes up by 20.8% (table 1) and 17.2% (table 2).The average number of patients in the system also goes up. This is important from the revenue point of view.

In order to decide if the addition of an extra bed would be viable, this increase in revenue would have to be compared against the cost of setting up a new bed as well as increase in other recurring expenditure (eg. salary). The other performance measures also have shown substantial improvement with the increase in bed capacity.

## CONCLUSIONS

In this paper, we have presented explicit results for the M/M/c/c queuing system assuming that customers are of reneging type. A few re-designed performance measures have also been presented. A numerical example using these results has been described. Since reneging is a commonly observed phenomenon it is our belief that results presented in this work will be of use to practitioners of queuing theory. One can obtain results of the traditional M/M/c/c models by substituting $\nu=0$ in our results. We are aware of the fact that it is possible to derive steady probabilities of the queuing system using birth and death model. However, we have presented the steady state equations as those helps us to derive closed form expressions for mean and variance of number of customers in the system. As a matter of fact, the same procedure can be extended to derive other moments of the steady state distribution. The limitations of this work stem from the Markovian assumptions. Extension of our results for general distribution is a pointer to future research.

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## APPENDICES

**Derivation of P´(1) under R_EOS:**

From equation (3.2) we have

$$\lambda p_{n-1} + (n+1)(\mu+\nu)p_{n+1} = \lambda p_n + n(\mu+\nu)p_n \qquad n=1,2,3,..c-1$$

Now multiplying both sides of the equation by $s^n$ and summing over n

$$\lambda s\sum_{n=1}^{c-1} p_{n-1}s^{n-1} + \frac{1}{s}\sum_{n=1}^{c-1}(n+1)(\mu+\nu)p_{n+1}s^{n+1} = \lambda\sum_{n=1}^{c-1} p_n s^n + \sum_{n=1}^{c-1} n(\mu+\nu)p_n s^n$$

$$\Rightarrow \lambda s\sum_{n=1}^{c-1} p_{n-1}s^{n-1} - \lambda\sum_{n=1}^{c-1} p_n s^n = \sum_{n=1}^{c-1} n(\mu+\nu)p_n s^n - \frac{1}{s}\sum_{n=1}^{c-1}(n+1)(\mu+\nu)p_{n+1}s^{n+1}$$

$$\Rightarrow \lambda s\left\{p_0 s^0 + p_1 s^1 + p_2 s^2 + p_3 s^3 + \dots p_{c-2}s^{c-2}\right\} -$$
$$\lambda\left\{p_1 s^1 + p_2 s^2 + p_3 s^3 + \dots p_{c-1}s^{c-1}\right\}$$
$$= \left[(\mu+\nu)p_1 s^1 + 2(\mu+\nu)p_2 s^2 + \dots + (c-1)\{\mu+\nu\}p_{c-1}s^{c-1}\right] -$$
$$\frac{1}{s}\left[2(\mu+\nu)p_2 s^2 + 3(\mu+\nu)p_3 s^3 + \dots + c\{\mu+\nu\}p_c s^c\right]$$
$$\Rightarrow \lambda s\left\{p(s) - p_{c-1}s^{c-1} - p_c s^c\right\} - \lambda\left\{p(s) - p_0 - p_c s^c\right\}$$
$$= s(\mu+\nu)\left(p_1 s^0 + 2p_2 s^1 + 3p_3 s^2 + \dots + (c-1)p_{c-1}s^{c-2}\right) -$$
$$(\mu+\nu)\left(2p_2 s + 3p_3 s^2 + \dots + cp_c s^{c-1}\right)$$

$$\Rightarrow \lambda sp(s) - \lambda p_{c-1}s^c - p_c s^c - \lambda p(s) + \lambda p_c s^c + \lambda p_0$$
$$= s(\mu+\nu)\left\{p'(s) - cp_c s^{c-1}\right\} - (\mu+\nu)\left\{p'(s) - p_1\right\}$$

$$\Rightarrow (\mu+\nu)p'(s) - s(\mu+\nu)p'(s) = (\mu+\nu)p_1 - c(\mu+\nu)p_c s^c$$
$$-\lambda sp(s) + \lambda p(s) + \lambda p_{c-1}s^c + \lambda sp_c s^c - \lambda p_c s^c - \lambda p_0$$
$$\Rightarrow (\mu+\nu)(1-s)p'(s) = \lambda(1-s)p(s) - \lambda(1-s)p_c s^c$$
$$\Rightarrow p'(s) = \frac{\lambda}{(\mu+\nu)}\left\{p(s) - p_c s^c\right\}$$

Therefore, $p''(s) = \frac{\lambda}{(\mu+\nu)}\left[\frac{\lambda}{(\mu+\nu)}\left\{P(s) - p_c s^c\right\} - c p_c s^{c-1}\right]$

Now

$$\lim_{s\to 1-} p'(s) = \lim_{s\to 1-}\left[\frac{\lambda}{(\mu+\nu)}\left\{p(s) - p_c s^c\right\}\right]$$

$$\Rightarrow p'(1) = \frac{\lambda}{(\mu+\nu)}\left\{p(1) - p_c\right\} \tag{A.1}$$

$$= \frac{\lambda}{(\mu+\nu)}(1 - p_c)$$

and

$$p''(1) = \frac{\lambda}{(\mu+\nu)}\left[\frac{\lambda(1-p_c)}{(\mu+\nu)} - c p_c\right] \tag{A.2}$$